Logarithms Practice Quiz Solutions

27 %

Name: key

1. Consider the graph of the exponential function $f(x) = 2^x - 3$. Down 3
   (a) Describe the characteristics of the function, with respect to the domain & range, intercepts, and asymptote(s).

   Domain: $\{\mathbb{R}\}$ 0.5 Range: $\{y > -3\}$ 0.5 Equation of Asymptote: $y = -3$ (H.A.) 0.5

   (b) State the domain, range, intercept, and asymptote for the **inverse** of $f(x)$.

   Domain: $\{x > -3\}$ 0.5 Range: $\{\mathbb{R}\}$ 0.5 Equation of Asymptote: $x = -3$ (V.A.) 0.5

   (c) Determine the **equation of the inverse** function. 1

   $y = 2^x - 3 \xrightarrow{\text{inverse}} x = 2^y - 3$

   $x + 3 = 2^y$

   $y = \log_2(x+3)$ Log Form

2. For the function $y = -5\log_3(x+3)$ state the indicated characteristics. 4

   Domain: $\{x > -3\}$ 0.5 Range: $\{\mathbb{R}\}$ 1 Equation of Asymptote: $x = -3$ V.A. 0.5 Any x intercepts: $x = -2$ 0.5 Any y intercepts: $y = -5$ 0.5

   Algebraically determine the intercepts, show your work here. 1

   • For x-int set $y = 0$:

   $\frac{0}{-5} = \frac{-5\log_3(x+3)}{-5}$

   $0 = \log_3(x+3)$

   $3^0 = x + 3 \Rightarrow x = -2$

   3^0 is 1, so $1 - 3 = -2$

   • For y-int set $x = 0$:

   $y = -5\log_3(0+3)$

   $= -5\log_3(3)$

   $= -5(1)$ $3^1 = 3$

   $y = -5$

3. Express in logarithmic form: $12^2 = 144$ 1 2

   $\log_{12} 144 = 2$

4. Express in exponential form: $log(x+3) = y$ 1

   $10^y = x + 3$ common Log/ BASE 10

5. Evaluate *without using a calculator*: You must show all steps / reasoning 2

   (a) $log_5 125 = 3$ 1

   $5^x = 125$ convert to exp form

   $5^x = 5^3$ Equate bases

   $x = 3 \leftarrow$ value of LOG!

   (b) $log_{1/3} 27 = -3$ 1

   $\left(\frac{1}{3}\right)^x = 27$

   $\left(\frac{1}{3}\right)^x = \left(\frac{1}{3}\right)^{-3}$

   $x = -3$

6. Solve each equation. *(Show all steps / justification)* 3

   (a) $log_3 x = 4$ 1

   Convert to exp-form

   $3^4 = x$

   $x = 81$

   (b) $log_x\left(\frac{1}{8}\right) = -3$ 1

   $x^{-3} = \frac{1}{8}$

   Get exponent of x to "1"...

   $(x^{-3})^{-\frac{1}{3}} = \left(\frac{1}{8}\right)^{-\frac{1}{3}}$

   $x = \left(\frac{8}{1}\right)^{\frac{1}{3}}$

   $x = \sqrt[3]{8} \Rightarrow x = 2$

   (c) $log_2\sqrt{32} = x$ 1

   $2^x = 32^{\frac{1}{2}}$

   $2^x = (2^5)^{\frac{1}{2}}$ $= x$

   $2^x = 2^{5/2}$

   $x = 5/2$

   (or 2.5)

7. Simplify (show all steps) to evaluate ... $= 4 + 3(\log_7 49) + (-4) + (1) + (0)$

   $= 4 + 3(2) - 4 + 1 \rightarrow = 7$

8. Use the laws of logarithms to simplify (write as a single log) and then evaluate each expression. 1 2

   (a) $log_{12}24 - log_{12}6 + log_{12}36$ 1

   $= \log_{12}\frac{24 \cdot 36}{6}$

   $= \log_{12} 144$

   (b) $log_2 72 - \frac{1}{2}(log_2 3 + log_2 27)$

   $= \log_2 72 - \frac{1}{2}(\log_2 81)$

   $= \log_2 72 - \log_2 81^{1/2}$

   This is $\sqrt{81}$,

   $= \log_2 \frac{72}{}$

9. If $log3 = P$ and $log5 = Q$, write an algebraic expression in terms of P and Q for each:

(a) $log15$ $= \log(3 \cdot 5)$

$= \log 3 + \log 5$

$= P + Q$

/1

(b) $log\frac{25}{\sqrt{3}}$ $= \log 25 - \log \sqrt{3}$

$= \log 5^2 - \log 3^{\frac{1}{2}}$

$= 2\log 5 - \frac{1}{2}\log 3$

$= 2Q - \frac{1}{2}P$

/1

/2

10. If $\boldsymbol{log x = 4}$, evaluate:

(a) $log(100\,x)$

Split into two logs:

$= \log 100 + \log x$ ← this is 4!

$(10^? = 100?)$

$= 2 + 4 \Rightarrow = 6$

(b) $log(\frac{\sqrt{x}}{1000})$ $= \log \sqrt{x} - \log 1000$

$= \log x^{1/2} - \log 1000$

$= \frac{1}{2}\log x - \log 1000$

$= \frac{1}{2}(4) - 3 \Rightarrow = -1$

/2

11. Solve each equation algebraically.

(a) $8^{3x+4} = 4^{x-9}$

$(2^3)^{3x+4} = (2^2)^{x-9}$

$2^{9x+12} = 2^{2x-18}$

$9x + 12 = 2x - 18$

$7x = -6$

$x = -6/7$

(b) $\log_2 x - \log_2 3 = 5$

$\log_2 \frac{x}{3} = 5$

$2^5 = \frac{x}{3}$

$x = 32 \cdot 3$

$x = 96$

/2

12.

$= \log x^2 - \log z^{1/2} + \log y^3$

$= \log \frac{x^2 y^3}{z^{1/2}}$ ← $\sqrt{z}$

/2

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